Math 10A with Professor Stankova
Quiz 8; Wednesday, 10/18/2017
Section #106; Time: 10 AM
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Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** When integrating by parts, if we set $dv = 2xdx$, then we need to set $v = x^2$.

> **Solution:** Now matter which antiderivative you choose, you will get the same answer.

2. **TRUE** False Simpson's method always gives the exact answer when integrating a cubic function.

> **Solution:** As we saw in class, if the 4th derivative of f is 0, then your bound for Simpson's method gives 0. This means that the approximation will give the exact answer. For cubics, the 4th derivative is always 0.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (7 points) Integrate $\int \cos(\sqrt{x})dx$.

> **Solution:** First we u sub to get $u = \sqrt{x}$ so $du = \frac{1}{2\sqrt{x}}dx$ and $dx = 2\sqrt{x}du = 2udu$ Then this integral becomes
>
> $$\int \cos(\sqrt{x})dx = \int 2u\cos(u)du.$$
>
> We can integrate by parts here by setting $r = 2u$ and $dt = \cos(u)du$ so $dr = 2du$ and $t = \sin(u)$. Thus, we get that the integral is
>
> $$2u\sin(u) - \int 2\sin(u)du = 2u\sin(u) - 2(-\cos(u)) + C = 2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C.$$

(b) (3 points) What is the smallest number of intervals n you need to use in order to guarantee that the trapezoid approximation of $\int_0^1 \frac{x^3}{6}dx$ is within $\frac{1}{12\cdot 101}$. (The error bound using trapezoid approximation is $\frac{K_2(b-a)^3}{12n^2}$.

Solution: First we need to calculate $K_2 = \max_{[1,3]} |(\frac{x^3}{6})''|$. The first derivative is $\frac{x^2}{2}$ and the second is x so the maximum of the absolute value is at $x = 1$ since the function x is always increasing. Therefore, $K_2 = 1$. Thus, we have that

$$\frac{1}{12 \cdot 101} = \frac{1(1-0)^3}{12n^2} \implies n^2 = \frac{12 \cdot 101}{12} = 101.$$

Therefore $n = \sqrt{101}$. But, we want the smallest number of intervals and so we need to take the ceiling. The ceiling of $\sqrt{101}$ is 11 so $N = 11$.